Simulation of acoustic waves in a fluid contained in a nuclear tank - Fluid-Structure Coupling

Hanna CHETOUANE

August 24, 2025

UFR of Mathematics and Informatics - University of Strasbourg

M1 CSMI Internship - Avnir Energy

Supervisors: Thomas LE GALL, Christophe PRUD'HOMME

I. Introduction

Introduction

- 2 months M1 CSMI internship in Villeurbanne, Avnir Energy
- Collaboration with Rayen TLILI (M2 CSMI internship)

CSMI-Avnir Energy collaboration for 2 years: Sasha ALIDADI HERAN, Rayen TLILI, Marie SENGLER

→ **Main goal:** Develop a digital tool for monitoring and maintaining nuclear power plant tanks (defect detection and preventive maintenance)

Avnir Energy

- Founded in 2015, approximately 30 employees
- **Fields:** nuclear, oil and gas, hydraulics, renewable energies, civil engineering
- **Specialization:** vibration monitoring (vibration, acoustics, ultrasound, radiation)
- **Partners:** CEA (applied research in nuclear engineering), Sonorch Technologies (acoustics and non-destructive testing), research laboratories (INSA Lyon, CEMOSIS Strasbourg)

Avnir Energy - Temporal Reversal

Avnir Energy's flagship method: **Temporal Reversal**

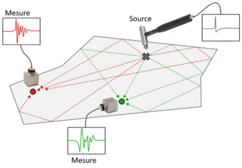


Figure: Learning phase

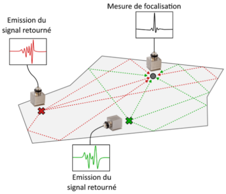
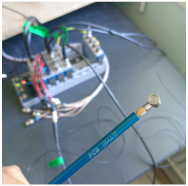


Figure: Signal reinjection phase



Advantages: accurate and early detection of defects and anticipation of their evolution

My objective: provide data for the learning phase (numerical data on the solid and fluid following an impact on the structure).

Internship Objectives

Develop a digital tool to simulate the **propagation of acoustic waves** in a fluid at rest inside a nuclear power tank, following an **impact on the outer wall**

→ in **C++** with the **Feel++** library

- Implementation of the wave equation and the elasticity equation
- Implementation of fluid-structure coupling (via fluid pressure and solid velocity)
- Test with realistic physical parameters and boundary conditions
- Simplified tank geometry (2D, 3D)

Mathematical and numerical tools

- **Finite element method (FEM):** discretization of space to numerically solve partial differential equations (PDEs)
- **Feel++:** C++ library for solving PDEs using FEM
- **GMSH:** geometry and meshing
- **Paraview:** visualization of results
- **Gaya:** parallel computing
- **Github:** collaborative management and code tracking

II. Theory

Physical modeling

Simulation of an impact on the outer wall of the tank $\Rightarrow$ two coupled phenomena:

- Deformation of the solid $\rightarrow$ **Elasticity equation**
  $\rightarrow$ displacement field, vector
- Wave propagation in the fluid $\rightarrow$ **Wave equation**
  $\rightarrow$ pressure field, scalar

Assumption: Small deformations and small perturbations over time: linear dynamic equations

Equation of elasticity

$$\rho_s \frac{\partial^2 u_s}{\partial t^2} - \nabla \cdot \sigma(u_s) = f_s \quad \text{in } \Omega_s \times [0, T]$$

where σ is the stress tensor:

$$\sigma(u_s) = \lambda(\nabla \cdot u_s)I + 2\mu\epsilon(u_s)$$

with I the identity matrix, ϵ the deformation tensor:

$$\epsilon(u_s) = \frac{1}{2}(\nabla u_s + \nabla u_s^T)$$

and $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$ the Lamé coefficients related to E the Young's modulus and ν the Poisson's ratio

Equation of elasticity

The different quantities are:

- u_s: displacement of the solid (vector field, m)
- ρ_s: density of the solid (kg/m^3)
- σ: stress tensor (Pa)
- ϵ: strain tensor (unitless)
- E: Young's modulus (Pa)
- ν: Poisson's ratio (Pa)
- f_s: source term (N/m^3)
- Ω_s: spatial domain of the solid (2D or 3D, m^2 or m^3)
- $[0, T]$: time domain (s)

Wave Equation

$$\frac{\partial^2 p}{\partial t^2} - c^2 \Delta p = f_f \quad \text{in } \Omega_f \times [0, T]$$

With:

- p: pressure field (scalar, in Pa)
- c: speed of sound in the fluid (m/s)
- f_f: source term (Ricker wavelet, Pa/s^2)
- Ω_f: spatial domain of the fluid (2D or 3D, m^2 or m^3)
- $[0, T]$: time domain (s)

Note: f_f is added in the case where we just simulate the wave equation (with coupling: $f_f = 0$)

Fluid-structure coupling strategy

Contribution of type **Neumann boundary conditions** on Γ_{fsi}

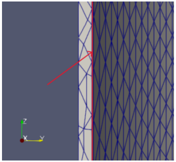


Figure: Fluide-structure Interface: Γ_{fsi}

- Equation of elasticity:

$$\sigma(u_s) \cdot n_s = -p n_s \quad \text{on } \Gamma_{fsi}$$

- Wave equation:

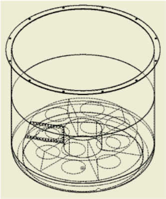
$$\frac{\partial p}{\partial n_f} = \rho_f \frac{\partial u_s}{\partial t} \cdot n_s \quad \text{on } \Gamma_{fsi}$$

Initial and boundary conditions

Initial conditions: System at rest:

$$u_s(t=0) = 0 \quad ; \quad p(t=0) = 0$$

Boundary conditions: tank held in place with its collar and open (no lid)



$\Rightarrow$ Dirichlet conditions:

$$u_s = 0 \quad \text{on } \Gamma_{s_{fixed}} \quad ; \quad p = 0 \quad \text{on } \Gamma_{top}$$

Complete System

$$
\begin{cases}
\rho_s \dfrac{\partial^2 u_s}{\partial t^2} - \nabla \cdot \sigma(u_s) = f_s \quad \text{in } \Omega_s \times [0, T] \\
\dfrac{\partial^2 p}{\partial t^2} - c^2 \Delta p = 0 \quad \text{in } \Omega_f \times [0, T] \\
u_s(t=0) = 0 \quad \text{in } \Omega_s \times \{0\} \\
p(t=0) = 0 \quad \text{in } \Omega_f \times \{0\} \\
u_s(x, y) = 0 \quad \text{in } \Gamma_{s_{fixed}} \times [0, T] \\
p(x, y) = 0 \quad \text{in } \Gamma_{top} \times [0, T] \\
\sigma(u_s) \cdot n_s = -p n_s \quad \text{in } \Gamma_{fsi} \times [0, T] \\
\dfrac{\partial p}{\partial n_f} = -\rho_f (\dfrac{\partial u_s}{\partial t} \cdot n_s) \quad \text{in } \Gamma_{fsi} \times [0, T]
\end{cases}
$$

Complete System

With:

- u_s: displacement of the solid (vector field, in m)
- p: pressure field (scalar, in Pa)
- $\sigma(u_s) = \lambda(\nabla \cdot u_s)I + 2\mu\epsilon(u_s)$: stress tensor (Pa)
- $\epsilon(u_s) = \frac{1}{2}(\nabla u_s + \nabla u_s^T)$: strain tensor (unitless)
- $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$ where E is Young's modulus (Pa) and ν is Poisson's ratio (Pa)
- ρ_s: density of the solid (in kg/m^3)
- ρ_f: density of the fluid (in kg/m^3)
- f_s: source term applied to the solid (in N/m^3)
- c: speed of sound in the fluid (in m/s)
- Ω_s: spatial domain of the solid (2D or 3D, in m^2 or m^3) with boundary Γ_{fsi}, $\Gamma_{s_{fixed}}$
- Ω_f: spatial domain of the fluid (2D or 3D, in m^2 or m^3) with boundary Γ_{fsi}, Γ_{top}
- $[0, T]$: time domain (in s)

Resolution with Feel++

Solving equations and fluid-structure coupling with Feel++. We provide:

- Time-discretized variational formulation
- Identification of test and trial functions and evaluation of functions
- Calculation of integrals of linear and bilinear terms

Feel++ manages:

- Matrix formulation and spatial assembly
- Solving the linear system at each time step

Variational formulation - Equation of elasticity

Let $u_s,\ v \in [H^1(\Omega_s)]^d$ $(d = 2$ or $3)$

$$\int_{\Omega_s} \rho_s \frac{\partial^2 u_s}{\partial t^2} \cdot v \, d\Omega_s - \int_{\Omega_s} (\nabla \cdot \sigma(u_s)) \cdot v \, d\Omega_s = \int_{\Omega_s} f_s \cdot v \, d\Omega_s$$

Green's formula:

$$\Leftrightarrow \int_{\Omega_s} \rho_s \frac{\partial^2 u_s}{\partial t^2} \cdot v \, d\Omega_s + \int_{\Omega_s} \sigma(u_s) : \nabla v \, d\Omega_s = \int_{\Omega_s} f_s \cdot v \, d\Omega_s + \int_{\partial\Omega_s} (\sigma(u_s) \cdot n_s) \cdot v \, d\partial\Omega_s$$

$$\Leftrightarrow \int_{\Omega_s} \rho_s \frac{\partial^2 u_s}{\partial t^2} \cdot v \, d\Omega_s + \int_{\Omega_s} \sigma(u_s) : \nabla v \, d\Omega_s = \int_{\Omega_s} f_s \cdot v \, d\Omega_s - \int_{\Gamma_{fsi}} (p n_s) \cdot v \, d\Gamma_{fsi}$$

Variational formulation - Equation of elasticity

Temporal discretization: Newmark scheme with $\beta = \frac{1}{4}$ and $\gamma = \frac{1}{2}$ for an unconditionally stable scheme

$$\frac{\partial^2 u_s}{\partial t^2} = a_s^{n+1} \approx \frac{u_s^{n+1} - u_s^n - \Delta t v_s^n - \frac{\Delta t^2}{2}(1-2\beta) a_s^n}{\beta \Delta t^2}$$

Gives us in **explicit**:

$$\int_{\Omega_s} \frac{\rho_s}{\beta \Delta t^2} u_s^{n+1} \cdot v \, d\Omega_s = \int_{\Omega_s} \rho_s \frac{u_s^n + \Delta t v_s^n + \frac{\Delta t^2}{2}(1-2\beta) a_s^n}{\beta \Delta t^2} \cdot v \, d\Omega_s$$

$$+ \int_{\Omega_s} \left[-\sigma(u_s^n) : \nabla v + f_s \cdot v \right] d\Omega_s - \int_{\Gamma_{fsi}} (p^n n_s) \cdot v \, d\Gamma_{fsi}$$

Variational formulation - Wave equation

Let $p,\ q \in H^1(\Omega_f)$

$$\int_{\Omega_f} \frac{\partial^2 p}{\partial t^2} q \, d\Omega_f - \int_{\Omega_f} c^2 (\Delta p) q \, d\Omega_f = \int_{\Omega_f} f_f q \, d\Omega_f$$

Green's formula:

$$\Leftrightarrow \int_{\Omega_f} \frac{\partial^2 p}{\partial t^2} q \, d\Omega_f + \int_{\Omega_f} c^2 \nabla p \cdot \nabla q \, d\Omega_f = \int_{\Omega_f} f_f q \, d\Omega_f + \int_{\partial\Omega_f} c^2 \frac{\partial p}{\partial n_f} q \, d\partial\Omega_f$$

$$\Leftrightarrow \int_{\Omega_f} \frac{\partial^2 p}{\partial t^2} q \, d\Omega_f + \int_{\Omega_f} c^2 \nabla p \cdot \nabla q \, d\Omega_f = \int_{\Omega_f} f_f q \, d\Omega_f - \int_{\Gamma_{fsi}} c^2 \rho_f \left(\frac{\partial u_s}{\partial t} \cdot n_s\right) q \, d\Gamma_{fsi}$$

Variational formulation - Wave equation

Temporal discretization: Second-order Bdf

$$\frac{\partial^2 p^{n+1}}{\partial t^2} \approx \frac{p^{n+1} - 2p^n + p^{n-1}}{\Delta t^2}$$

Gives us in **explicit**:

$$\int_{\Omega_f} \frac{p^{n+1}}{\Delta t^2} q \, d\Omega_f = \int_{\Omega_f} \frac{2p^n - p^{n-1}}{\Delta t^2} q - c^2 \int_{\Omega_f} \nabla p^n \cdot \nabla q \, d\Omega_f$$

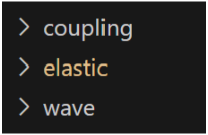
$$+ \int_{\Omega_f} f_f^n q \, d\Omega_f - \int_{\Gamma_{fsi}} c^2 \rho_f (v_s^{n+1} \cdot n_s) q \, d\Gamma_{fsi}$$

III. Implementation

Structure

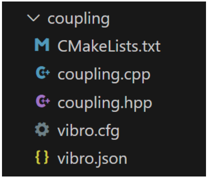
C++ code in the form of classes:

- class Elasticity
- class Wave
- class Coupling



Related files:

- .geo file: mesh file
- .json file: physical and numerical parameters
- .cfg file: resolution parameters



Solving equations in monolithic

- **Constructor:**
  - **readJson()**: reads parameters (.json file)
  - **initialize()**: creates the objects needed for the simulation (function space, time scheme, initial solution, test function, linear and bilinear forms)
- **Solve():** adds integrals (linear and bilinear forms), solves the system with a linear solver
- **run():** main time loop and executes the equation in monolithic mode $\rightarrow$ calls Solve(), Export() and moves on to the next time step

Start the simulation: create the equation object (with mesh and export) and call the run() function

Solving coupled equations

Picard iterations: reinforces fluid-structure coupling

Algorithm: Picard iterations over a time step

Data: u_s^n, p^n, k_{max}, ϵ
Result: u_s^{n+1}, p^{n+1}
$u_s^{n,0} \leftarrow$ Elastic.getCurrentDisp();
$v_s^{n,0} \leftarrow$ Elastic.getCurrentVelocity();
$p^{n,0} \leftarrow$ Elastic.getCurrentPressure();
$k \leftarrow 0$;
while *not converged and* $k < k_{max}$ **do**
  $k \leftarrow k + 1$;
  Elastic.Solve(project(Ω_s, Γ_{fsi}, $p^{n,k-1}$));
  $v_s^{n,k} \leftarrow$ Elastic.getCurrentVelocity();
  Wave.Solve(project(Ω_f, Γ_{fsi}, $v_s^{n,k}$));
  $u_s^{n,k} \leftarrow$ Elastic.getCurrentDisp();
  $p^{n,k} \leftarrow$ Elastic.getCurrentPressure();
  if $\|u^{n,k} - u^{n,k-1}\| < \epsilon$ *and* $\|p^{n,k} - p^{n,k-1}\| < \epsilon$ **then**
    converged $\leftarrow$ *true;*
  Elastic.Update();
  Wave.Update();
  return u_s^{n+1}, p^{n+1};

Start the simulation: call run() of the Coupling object

IV. Simulation

Parameters

Fluid modeling: Water
$c = 1480$ m/s, $\rho_f = 1000$ kg/m^3

Solid modeling: Stainless steel
$E = 2.1 \times 10^{11}$ Pa, $\nu = 0.3$ Pa, $\rho_s = 7870$ kg/m^3

Source function: Ricker Wavelet
$f(x, y, t) = A\psi(t)$ with $\psi(t) = -sin(2\pi f_c t)exp(-5(f_c t - 2)^2)$ and
$A = 10^3$ Pa, $f_c = 10^5$ Hz

Solution scheme

Elasticity equation: Implicit $\rightarrow$ no CFL conditions
Wave equation: Explicit $\rightarrow$ CFL condition:

$$\Delta t \leq \frac{h}{c\sqrt{d}}$$

With:

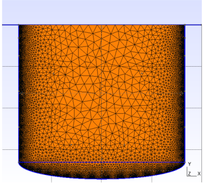
- h: space step
- c: speed of sound in the fluid
- d: dimension (2D or 3D)

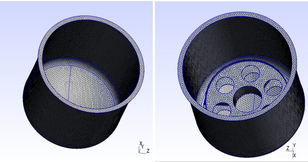
Simulations: $\Delta t = 5 \times 10^{-7}$, $h = 0.01$ to 0.003

Geometries

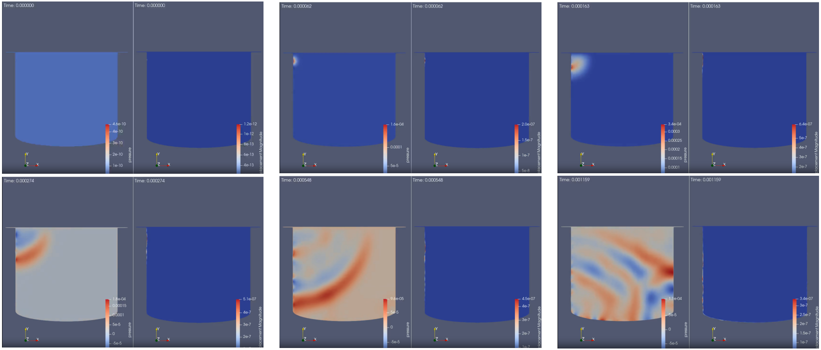
2D and 3D geometries of a simplified tank:

- adaptive mesh
- segment/surface for the applied force
- measurement points for extracting results

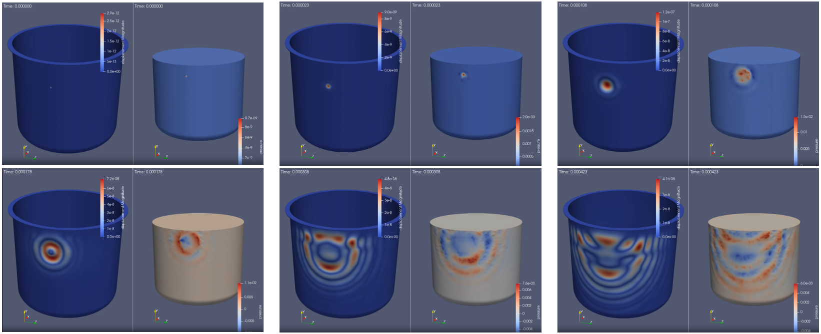




Results - 2D



2D visualizations of fluid-structure coupling - pressure on the left, solid displacement on the right



3D visualizations of fluid-structure coupling - pressure on the right, solid displacement on the left

Calculation Time - 2D

Simulation 2D: on 128 computing cores, on one time step:

Mesh	Solve Elasticity [s]	Solve Wave [s]	Coupling: time per step [s]
12 mm	0.015202	0.0163906	2.57761

- Solve Elasticity time and Solve Wave time close: Wave on explicit (faster) but calculation on many more elements
- Coupling time: two Picard iterations, projection on the other space, export

Calculation Time - 3D

Simulation 3D: on 128 computing cores, on one time step:

Mesh	Solve Elasticity [s]	Solve Wave [s]	Coupling: time per step [s]
12 mm	0.1319	0.0651355	3.27349
6 mm	0.311	0.147052	5.31692
3 mm	0.621	0.482818	17.0882

- Many more elements in 3D than in 2D
- Mesh partitioning to exploit parallelism
- Space refinement: greater precision but longer calculation times $\rightarrow$ find a compromise

V. Conclusion

Conclusion

Objectives achieved:

- Development of a vibro-acoustic toolbox to address the fluid-structure problem, usable by Avnir Energy
- Simulation with a simplified tank, in water with a stainless steel wall
- Results validate our approach

Perspectives

→ Initial working base for further development:

- Explicit elasticity equation: reduces computation time (still not stable even with $\Delta t = 10^{-9}$)
- Benchmarking studies: more advanced analysis of calculation time, quantifying the impact of numerical choices on calculation time and accuracy
- Addition of an ALE map: taking into account mesh deformation (tank wall)
- Docker image: making our code executable only by creating the environment necessary for compilation (to be changed)
- More realistic boundary conditions and tank geometry
- Exporting measurements to a point: work in progress
- Analytical validation
- Comparison of results with the FSI toolbox and experimental data from Sonorch Technologies